Transitional Buckling Model for Active Bending Effect in Pole Vault

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1 INTRODUCTION

Tool-use enables humans to accomplish tasks that cannot be realized solely with the human body. In the robotic motion, tools can be used to extend the robot's body, allowing it to achieve tasks that would be otherwise impossible to complete [1]. That is to say, skillful tool-use dramatically improves the robot's performance of locomotion. In those skill, a particularly interesting task is the pole vault, in which the athlete's movement dynamically changes depending on the way the pole is manipulated.

It is important for improving the vaulting performance that the athlete suspending from the pole actively manipulates the pole. In fact, it is known that the athlete's total energy when crossing the bar can exceed 120% of the initial energy at takeoff [2]. Many previous studies have measured the athlete's mid-flight behavior. For example, Frère *et al.* took electromyograms of the muscles in upper limbs and analyzed the role of each muscle in the overall action [2]. Other studies such as McGinnis *et al.* used inverse kinematics to calculate the moment exerted by the athlete mid-flight [3]. These studies have focused on measuring the athlete's behavior. However, they have not explicitly addressed the how the athlete's manipulation of the pole contribute to his overall performance.

Therefore, in this study, we try to analyze the active bending effect to the vaulting performance proposing "Transitional Buckling Model". This proposed model accounts for the athlete's active bending on the pole. In addition, we find out how the robot should actuate the pole. Accordingly, we present one of the way to skillfully use the flexible and complex tools.

2 MODEL

2.1 Active Bending of the Pole

In a pole vault, it is known that the athlete's body movements have a significant impact on the pole. Based on previous works which use EMG signals from upper limbs [2] or use inverse kinematics [3] to analyze the athlete's actions, it is known that an experienced athlete bends his pole as follows (Fig.1).

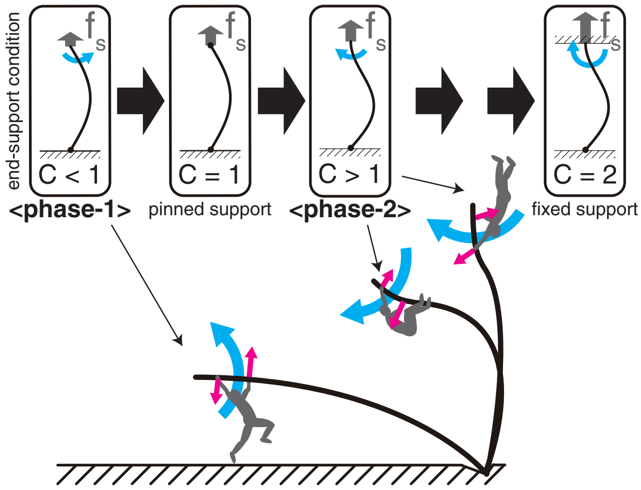

Figure 1: Transitional Buckling Model. Pink linear arrows are forces from arms. Blue curved arrows are input bending moments from arm forces.

Phase 1: Pole-plant and pole-bending phase
By applying an upward force on the lower handgrip of the pole, the athlete bends the pole in such a way as to increase the pole curvature.

Phase 2: Pole-straightening phase
By doing a handstand mid-flight, the athlete bends the pole in such a way as to reduce the pole curvature.

It has enough studied that how the athlete action, but not enough studied that how the action affect vaulting performance. Thus, there is a need to model a system including both the human and the pole.

2.2 Transitional Buckling Model for Active Bending

First, we modeled the overall pole vault motion illustrated in Fig.2. It was quite difficult to model the system containing flexible pole so that we modeled the system by Euler buckling model. Euler buckling model can treat the force exerted by the flexible pole as a simple constant force and is generally used for analysis of the pole vault. Besides, we treated vaulter and vaulter's

Table 1: Simulation parameter

parameter	description	experiment1	experiment2
θ	elevation angle of mass	variable	$\rightarrow$
l	displacement from pole lenglth	variable	$\rightarrow$
θ_0	initial angle	$\sin^{-1}(l/h_0)$	$\rightarrow$
l_0	pole length	3-5[m]	$\rightarrow$
h_0	initial height	2.0[m]	$\rightarrow$
v_0	initial velocity	6-9[m/s]	9[m/s]
m	mass of vaulter	80[kg]	$\rightarrow$
g	acceleration of gravity	9.8[m/s^2]	$\rightarrow$
E	Young's modulus	70[GPa]	$\rightarrow$
I	second moment of area	5[cm^4]	$\rightarrow$
f_s	exerted force from the pole	-	$\rightarrow$
u	input bending moment	-	$\rightarrow$

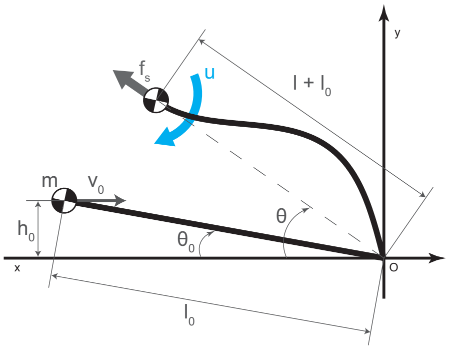

Figure 2: Pole suspending one point mass model.

motion as the mass point and bending moment. Therefore, the equation of motion is represented as:

$$\frac{d}{dt}\begin{pmatrix}\theta\\ \dot{\theta}\\ l\\ \dot{l}\end{pmatrix} = \begin{pmatrix}\dot{\theta}\\ -\frac{1}{l+l_0}(2\dot{l}\dot{\theta}+g\cos\theta)\\ \dot{l}\\ -(l+l_0)\dot{\theta}^2+g\sin\theta\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 0\\ \frac{f_s}{m}\end{pmatrix}, \tag{1}$$

$$\text{where} \quad f_s = C\frac{\pi^2 EI}{l_0^2} = (\text{const.}).$$

Here, C is the end-support condition coefficient, E is Young's modulus, and I is second moment of area. The mass point has horizontal initial velocity v_0 and the pole applies the exerted force f_s on the mass point. The exerted force f_s can be represented by Euler buckling load. According to the Euler buckling model, as long as the end-support condition of the pole remains constant, f_s remains constant regardless of pole deformation. In the Euler buckling model, the coefficient C is determined by the end-support condition (Fig.1). The exerted force f_s is constrained by the coefficient C.

Second, we represent active bending effect presented in Sec.2.1 as the extended Euler buckling model. Most previous pole vault models have treated both the top and bottom end of the pole as pinned support, and C has been usually set to constant as $C = 1$ [4] [5]. In contrast, our proposed model also treats the bottom end of the pole as pinned support, while it differently treats the top end of the pole as variable support transitioning as a function of input bending moment u (Fig.1). Thus C is set to variable $C = C(u)$. The exerted force f'_s is represented as:

$$f'_s = C(u)\frac{\pi^2 EI}{l_0^2}. \tag{2}$$

Therefore, to substitute f'_s for f_s in Eq.(1) properly accounts for bending moment influence. We call this proposed model the Transitional Buckling Model (TBM). The relationship between the active bending action and $C(u)$ is as follows.

Phase 1: $C(u) < 1$
Force received from the pole f'_s is small, so that the pole can be bent with a larger curvature than in $C = 1$.

Phase 2: $C(u) > 1$
Force received from the pole f'_s is large, so that the mass point can reach a greater height.

3 SIMULATION EXPERIMENTS

We focused on the advantage of input bending moment. To compare that we conducted follow experiment. First, we experimentally compared the original buckling model treating C as constant and the Transitional Buckling Model treating $C(u)$ as variable. Second, in Transitional Buckling Model, we experimentally explored the vaulting performance shifting the timing and change rate of the transition of $C(u)$.

3.1 Experimental Setup

We simulated the pole vault by use of numerically-solving ordinary differential equation eq.(1) with a step time of 1[ms]. Simulation parameters was determined

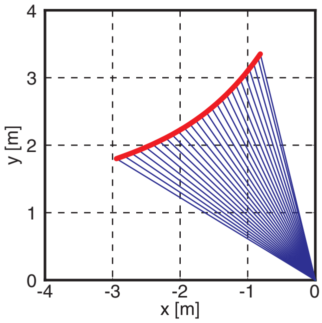

Figure 3: Trajectory of the mass point while the mass point contacts with the pole. Red heavy line is the trajectory of the mass point. Blue thin line is the segment from origin to the mass point. $l_0 = 3.5$[m], $v_0 = 9.0$[m/s].

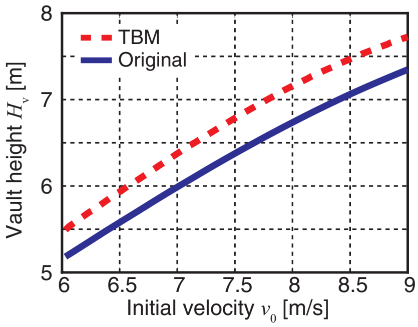

Figure 4: Vault height at each initial velocity. 'original' does not include active bending, and 'TBM' includes it.

referring to an actual athlete's data (Table1). Vault height H_v is a maximum height of the trajectory of the mass point. The mass point lets go of the pole when the pole straightens, so that it takes off in a projectile motion.

3.2 Experiment1: Active Bending Effect

In original Euler buckling model, C is constant as $C = 1$. In Transitional Buckling Model (TBM), $C(u)$ is linearly changed from $C_0 = 0.8$ to $C_1 = 1.2$. We changed initial velocity from 6.0[m/s] to 9.0[m/s]. In TBM, vault height H_v at each initial velocity was larger than in the original model (Fig.4). Here, H_v is a maximum height of the trajectory of the mass point. The mass point lets go of the pole when the pole straightens, so that it takes off in a projectile motion. Thus, input bending moment affects the vaulting performance.

3.3 Experiment2: How to Change Input Moment

In TBM, we analyzed the effect of how $C(u)$ changes from C_0 to C_1. Vault height H_v map show that H_v took a high value around $\dot{l} = 0$ and sharp increase area

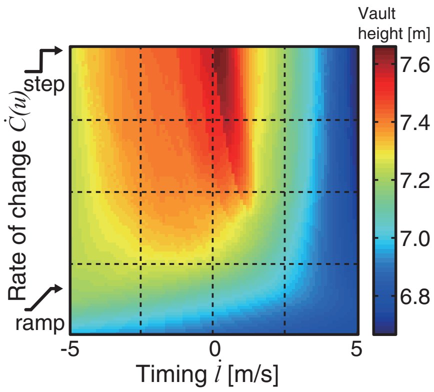

Figure 5: Vault height map by the transition timing and the rate of change with end-support condition coefficient $C(u)$. Horizontal axis is the transition timing normalized by $\dot{l}$. When $\dot{C}(u)$ is large, input bending moment is like a step input.

(Fig.5). The point of $\dot{l} = 0$ is the point of minimum l, which is the timing at which the pole is maximally bent. Therefore, the robot should change the direction of the bending moment like a step input at the time when the pole is maximally bent.

4 CONCLUSION

In this paper, we analyzed the active bending effect to vaulting performance proposing a model that the pole's end-support condition varies with input bending moment. We showed that input bending moment improved the vaulting performance. In addition, we analyzed the best timing and the change rate at which the bending moment direction to improve the vaulting performance. We found that the robot should change the direction of bending moment like a step input at the time when the pole is maximally bent. Accordingly, we inspire the way of skillfully using complex flexible tools.

In the future, we will implement the above model in a control theory of a pole vaulting robot.

5 OPEN QUESTIONS

The equation for input bending moment u and end-support condition coefficient $C(u)$ is still an open question.

6 ACKNOWLEDGEMENT

We thank Yuko Hara for her valuable comments on the draft of the manuscript. This work has been supported by MEXT/JSPS KAKENHI Grant Number 22240015.

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