

## Chapter 1

# Quantification of Emergent Behaviors Induced by Feedback Resonance of Chaos

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We address the issue of how an embodied system can autonomously explore and discover the action possibilities inherent to its body. Our basic assumption is that the intrinsic dynamics of a system can be explored by perturbing the system through small but well-timed feedback actions and by exploiting a mechanism of feedback resonance. We hypothesize that such perturbations, if appropriately chosen, can favor the transitions from one stable attractor to another, and the discovery of stable postural configurations. To test our ideas, we realize an experimental system consisting of a ring-shaped mass-spring structure driven by a network of coupled chaotic pattern generators (called coupled chaotic fields). We study the role played by the chaoticity of the neural system as the control parameter governing phase transitions in movement space. Through a frequency-domain analysis of the emergent behavioral patterns, we show that the system discovers regions of its state space exhibiting notable properties.

Keywords: Emergence, resonance, globally coupled maps, motor skill acquisition, degrees of freedom problem, wavelet transform, spectral bifurcation map, wavelet bifurcation map.

### 1.1 Introduction

How do infants explore their bodies and acquire motor skills? How do humans and other animals adapt to unexpected contingencies and opportuni-

ties in a dynamic and ever changing environment such as the real world? Or more in general, what are the mechanisms that allow a complex embodied system consisting of a multitude of coupled and potentially heterogenous elements to autonomously explore, discover, and select possibilities for action and movement? These are difficult issues whose answers despite intensive efforts still elude us. The main goal of this paper is to shed some light on how body dynamics might be explored. Exploration and emergence represent important first steps towards gaining further insights into how higher level cognitive skills are bootstrapped.

Nikolaus Bernstein was probably the first to address in a systematic way the question of how humans purposefully exploit the interaction between neural and body-environment dynamics to solve the complex equations of motion involved in the coordination of the large number of mechanical degrees of freedom of our bodies [1]. In the last decade or so, Bernstein's degrees of freedom problem has been tackled many times through the framework of dynamical systems (e.g. [3; 14]).

Such research has three important implications which are relevant for this paper. First, movements are dynamically soft-assembled by the continuous and mutual interaction between the neural and the body-environment dynamics. Second, embodiment and task dynamics impose consistent and invariant (i.e. learnable) structure on sensorimotor patterns. Third, when the neural dynamics of a system is coupled with its natural intrinsic dynamics, even a complicated body can exhibit very robust and flexible behavior, mainly as a result of mutual entrainment (e.g. neural oscillator based biped walking [13] and pendulation [5]).

In this paper, we pursue further the idea of a network of chaotic units used as a model for exploration of body dynamics [4]. One of the core features of our model is that it allows to switch between different attractors while maintaining adaptivity. We make two main contributions: The first one is the introduction of a mechanism of feedback resonance of chaos in our model. The second contribution is a set of tools for analyzing the resulting spatio-temporal dynamics. In the following section, we will present the three pillars on which our augmented model rests: (a) dynamical systems approach, in particular the notions of global dynamics and interaction dynamics; (b) mechanism of feedback resonance thanks to which the neural system tunes into the natural frequencies of the intrinsic dynamics of the mechanical system; and (c) concept of coupled chaotic fields which is responsible for exploration of the neural dynamics. We then introduce a set of methods used to analyze the spatio-temporal dynamics of the neural and mechanical system. Subsequently, we describe our experimental setup and report the outcome of our experiments. In the last section, we discuss

our results and conclude by pointing to some future work.

## 1.2 Model System

In this section, we introduce the three key elements of our model of exploration. We briefly describe dynamical systems theory, and the concepts of feedback resonance and coupled chaotic field.

### 1.2.1 *Dynamical Systems Approach*

Dynamic systems theory is a well-established framework primarily geared towards modeling and describing nonlinear phenomena that involve change over time [12; 15]. From a dynamical point of view, systems are typically described in terms of their evolution over time, their robustness against internal and external perturbations, number and type of attractors and repellers, as well as bifurcations, that is, qualitative changes of the dynamics of the system occurring at critical states.

A dynamical system – initialized in a particular stable attractor state and affected by noise – fluctuates irregularly inside it despite internal and external perturbations. The system cannot evolve to a new state until the perturbations reach a certain level triggering a transition to a new (possibly more stable) attractor. Here, we conceive of perturbations (internal and external ones) (a) as a means to explore and discover stable as well as unstable action possibilities of a mechanical system, and (b) as a mechanism of adaptation against environmental changes. Our approach is reminiscent of the process of chaotic itinerancy which can be defined as the chaotic transition dynamics resulting from a weak instability of the attractor landscape of the dynamical system [3].

By contrast, control theoretical approaches (e.g. [11]), including adaptive methodologies, are typically framed as abstract mathematical problems and avoid exploiting any nonlinear physical aspect in the control process such as interactions, perturbations, body dynamics, and entrainment.

### 1.2.2 *Feedback Resonance*

The second key element of our approach is feedback resonance [2; 8]. This mechanism indicates that small but timely feedback actions can dramatically affect the dynamics of a nonlinear system, e.g. turn chaotic motions into periodic ones. The rationale is that by having the feedback actions occur at a specific instant in time, it is possible either to entrain a system to the action or to destabilize it and induce a transition to a new behavioral

pattern. The phenomenon can be conceptualized as the energy-efficient excitation through resonance of the many degrees of freedom of a system. Resonance is pivotal because when the frequency of oscillation of the system matches its natural vibration frequency, the system absorbs injected oscillatory energy more effectively.

Action on the system using feedback resonance is described as:

$$F_i(t+1) = F_i(t) + \gamma x_i(t), \quad (1.1)$$

where  $F_i(t) \gg \gamma x_i(t)$  expresses the dynamics of the system (here, the force acting on motor  $i$  at time  $t$ ), and  $x_i(t)$  is the controlled variable (here, the neural excitation) scaled by a parameter  $\gamma$ . By exploiting resonance, the system can amplify the small perturbations and dramatically affect the global dynamics of the system inducing bifurcations and new postural configurations (in the case of a mechanical system). The resonant forces are used to transfer energy to new behaviors. The general idea is akin to the concepts of global dynamics and intervention introduced by [16].

The work done in physics has mainly focused on idealized pendular systems and chaotic models [2]. By contrast, our framework explicitly includes information about the morphology of the embodied system (i.e. distribution and type of actuators and sensors), the properties of the environment, as well as the coupling between body and neural system. The dynamics of the body embedded in the environment is exploited and natural resonant states are discovered.

### **1.2.3 Coupled Chaotic Field**

We used a chaotic pattern generator as an internal source of perturbation (external perturbations are gravity, impactive forces acting on the body, and so on). As a model of neural activity we chose the globally coupled map (GCM) which is a network of chaotic elements instantiating a minimal model that has local chaotic dynamics but also global interaction between the elements [3]. Although simple, such maps exhibit a rich and complex dynamics. A coupled chaotic field (CCF) is essentially a globally coupled map in which every chaotic element is connected to the embodied system through sensors (supplying the input) and motors (to which the output is relayed). In other words, the global coupling between the chaotic units is provided by the environmental interaction (see Fig. 1.1).

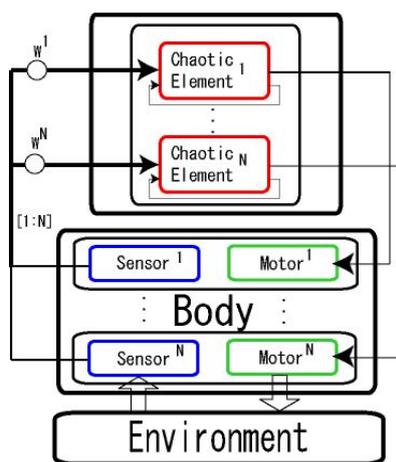


Fig. 1.1 Outline of our model.

Formally, the neural system can be described as follows:

$$x_i(t+1) = (1 - \epsilon)f_\alpha(x_i(t)) + \frac{\epsilon}{N} \sum_{j=1}^N f_\alpha(x_j(t)) \quad (1.2)$$

$$f_\alpha(x_i(t)) = 1 - \alpha x_i(t)^2 + \eta F_i(t) \quad (1.3)$$

$$1 - \alpha x_i(t)^2 \gg \eta F_i(t) \text{ with } \alpha \in ]0.0, 2.0] \text{ and } \epsilon \in ]0.0, 0.4]$$

The parameters  $\alpha$  and  $\epsilon$  are the two control parameters of the CCF, and  $f_\alpha(x)$  realizes the chaotic dynamics of the neural unit at time  $t$ . The variable  $\alpha$  determines the chaoticity of each neural unit and  $\epsilon$  controls the level of synchronization among the units. In our experiments, the only control parameter was the chaoticity. The level of synchronization was fixed to a small number ( $\epsilon < 0.05$ ) so as not to bias the emergence of synchronized and coherent states. In fact, rather than initializing the synchronization parameter to a specific value among the units and have a static coupling among the chaotic system, the coupling was dynamically altered through internal and external perturbations (here, the CCF was perturbed by the output of force sensors located in the joints). The coupling constant  $\eta$  (eq. 1.2.3) was selected empirically. Its value was small enough so as not too affect too much the intrinsic dynamics of the chaotic units. The output of the CCF was then fed – after appropriate scaling – to the motors (eq.1.1).

By letting neural and body-environment dynamics interact many instances of mutual entrainment among the two dynamical systems could be observed. This kind of mutual interaction has been called global entrainment, and has been hypothesized to generate stable and flexible movements in a self-organized manner despite unpredictable changes in environmental conditions [13].

### 1.3 Methods

In this section, we introduce the tools used to analyze the data generated in our experiments. The main purpose of our methods is to quantitatively assess behavioral patterns. We describe three methods: (1) the "Spectral Bifurcation Diagram", (2) the "Wavelet Transform", and (3) a novel visualization method inspired by methods 1 and 2, which we shall refer to as the "Wavelet Bifurcation Diagram."

The Spectral Bifurcation Diagram is a recently introduced method for investigating the qualitative changes of the dynamics of high-dimensional systems [7]. Essentially, it displays the power density spectrum of a system variable as a function of a control parameter of the system (the power spectra of the individual variables are superposed). The control parameter is a variable to which the behavior of the system is sensitive and that moves the system through different (attractor) states. The representation illustrates how the neural system affects the coordination among the different parts of the body in frequency space. This method allows to identify resonant states which are characterized by sharp frequency components, chaotic states having rather broad power spectra, as well as bifurcations, that is, abrupt transitions from one attractor state to another.

The second method used was the Wavelet transform [6]. In the Wavelet space, one variable of the system is projected onto a space spanning time and frequency in which it is possible to identify changes of behavior at different time scales, short-range as well as long-range temporal correlations. The Wavelet transform thus allows to analyze temporal bifurcations, and consequently the evolution of the dynamics of each unit of the embedded neural system.

Because we are also interested in understanding the spatial correlations between neural units while performing a certain movement, as well as the type of interactions at different spatio-temporal scales, we introduced a novel tool for the analysis of high-dimensional systems spanning frequency, time, and index of the neural unit. In this space, we are not only able to identify spatio-temporal correlations but also changes over time of each unit or groups of units as well as bifurcations in the dynamics of systems with

a large number of degrees of freedom. We refer to this method as Wavelet Bifurcation Diagram.

#### 1.4 Experiments

For our experiments we used a ring-shaped mass-spring robotic system composed of 10 prismatic elements connected by 10 force-controlled sliding joints (Fig. 1.2). Each element was connected to its two neighbours by compression springs. The spring-motor complex had 3 degrees of freedom (DOF), and the complete mechanical system had a total of 30 DOF. The neural and body dynamics were mutually coupled as explained previously (Eqs. 1.1 and 1.3). The robot was placed in a plane devoid of obstacles and of other external perturbations except gravity, ground reaction forces, and friction. We realized our ring-shaped robot using a physics-based simulator (see [10]). Table 1.1 shows the parameters we used for all our simulations. It is important to stress that despite its simplicity, our robot model can display a sufficiently large set of behaviors, and is therefore appropriate for investigating the mechanisms underlying the emergence of stable as well as as unstable behavioral patterns.

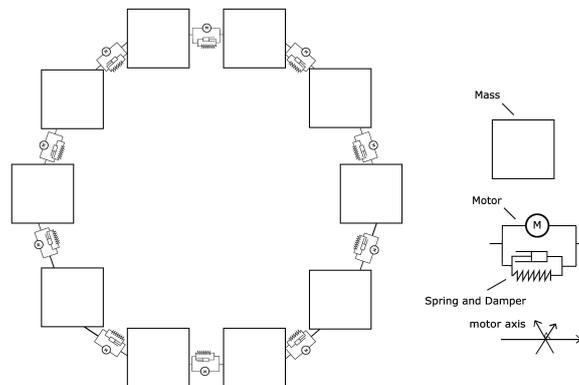


Fig. 1.2 Schematic representation of our robot model. The masses are prisms and are connected to each other by force-controlled compliant linear actuators. The system has 30 degrees of freedom.

Our analysis was mainly focused at understanding how the body dynamics evolved in time as a function of the chaoticity  $\alpha$  of the neural units. The initial value of  $\alpha$  was zero. For this value the output of the neural

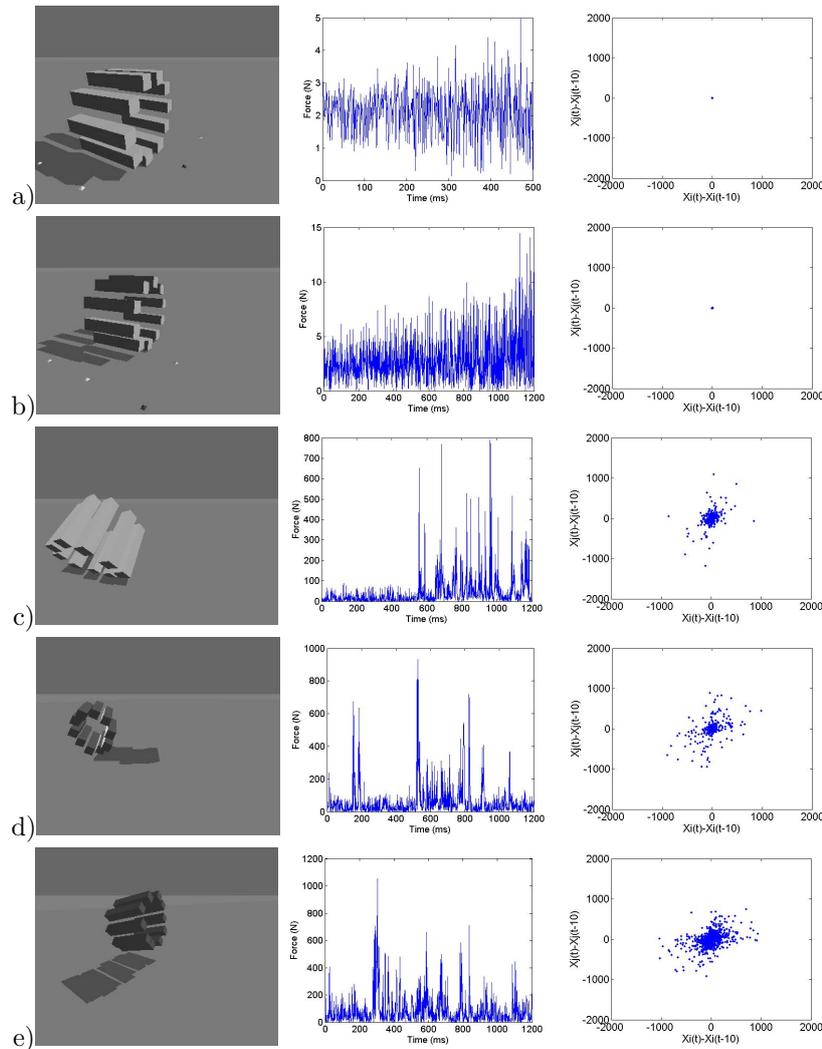


Fig. 1.3 Left: Snapshots of different postures for different  $\alpha$ ; (a) balance ( $\alpha = 0.05$ ), (b) rolling ( $\alpha = 0.10$ ), (c and d) quick and unstable movement ( $\alpha = 1.0$  and  $\alpha = 1.5$ ), and (e) uncoordinated movement pattern ( $\alpha = 1.9$ ). Center: Time series of one joint force pressure (unit: N). Right: Time-delayed phase portraits of two arbitrarily selected neighboring joints as a function of  $\alpha$ .

system was a constant, and the system did not move. By increasing by a small amount the chaoticity of the neural units ( $\alpha \in ]0.0, 0.1]$ ), the "ring" started to vibrate almost unperceivably at a very small spatial scale. After

$\gamma$	$\epsilon$	$\eta$	density	width	damping	stiffness
0.001	0.001	0.01	2.0	0.1	0.3	0.5

a certain amount of time, the seemingly random movements converged to a slow rhythmical rocking movement, that is, the system's dynamics had found a stable attractor. Figure 1.3 shows the phase space trajectories of two arbitrarily selected neighbouring joints for different values of  $\alpha$ . It plots  $X_i(t) - X_i(t - t')$  against  $X_j(t) - X_j(t - t')$  giving some information concerning the spatial correlations ( $X_i$  vs.  $X_j$ ) and the temporal correlations between joint variables ( $t' = 10$ ).

Up to a very specific level of chaoticity ( $\alpha = 0.097477$ ), the system strove a perfectly poised balanced posture, and did not have sufficient energy to start rolling. By slightly increasing the control parameter ( $\alpha = 0.1$ ), a phase transition occurred, and the system started to roll. Interestingly, for values of  $\alpha < 0.15$ , the system oscillated quite unpredictably between rolling and balancing. We hypothesize that the emergence of particular movement patterns depends on the presence or absence of entrainment between neural and body-environment dynamics.

For levels of neural chaoticity  $\alpha \in [0.15; 0.4]$ , novel behaviors and patterns of locomotion emerged. The neural system seemed to exploit the natural dynamics of the ring-shaped body to balance, rock, roll, accelerate and decelerate, and in rare occasions, even to jump. We suggest that the instabilities in the movements patterns were mainly caused by micro-scale perturbations acting on the neural dynamics with a consequent disruption of the entrainment between neural and body-environment dynamics and the emergence of new locomotion patterns.

In the range  $\alpha \in [0.4; 1.2]$ , the perturbations were larger and thus had a more pronounced effect on the system's dynamics. While still displaying coherence (that is, rather strong correlations between neighbouring segments of the ring), the movements were generally more complex and characterized by abrupt changes. Most notable was the high sensitivity of the system to internal (neural) and external perturbations (due to sensory feedback), and the emergence of a number of different behavioral patterns. The ring rolled quickly, accelerated, decelerated, and displayed many unstable postural configurations (such as balancing on an edge).

Finally, for  $\alpha$  in the interval  $[1.2; 2]$ , the system did not display any coherent or organized movement patterns. The activation levels of the chaotic system were too large to be influenced (perturbed) by sensory feedback causing a disruption of entrainment between neural and body-environment dynamics.

## 1.5 Analysis

One immediate implication of our experiments is that in an embodied system, the exploration of the space of possible coherent and stable postural modes is induced by the mutual adaptation between neural and body-environment dynamics. In other words, the neural system, the body, and the environment are all responsible for the emergence of particular movement patterns. Note that this global view contrasts with the one that sees only neural parameters responsible for exploration of the movement space.

### 1.5.1 *Analysis 1: Body movements*

The Spectral Bifurcation Diagram for varying levels of chaoticity is reproduced in Figure 1.4. Low levels of chaoticity ( $\alpha < 0.05$ ) are characterized by sharp peaks in the power spectral density of the force sensors located in the joints, and given a particular value of  $\alpha$  the resonance response is close to the one of a damped oscillator. The low frequency component around  $10\text{ Hz}$  dominates the interaction dynamics between the neural system and the ring. This frequency corresponds to the fundamental mode of the coupled system, that is, its eigen-frequency. For this frequency, the joints are highly synchronized and the system displays a high degree of coordination. A minimal amount of energy is required to move the system and to transfer energy to the different parts of the body.

When the chaoticity increases higher harmonics appear introducing discontinuities in the resonance response. The main resonance persists for all values of  $\alpha$  but we observe abrupt changes and bifurcations in the magnitude of other peaks. The new harmonic peaks are located at integer and fractional multiples of the first eigen-frequency [15]. The latter peaks are caused by small damped actions of the chaotic system and affect the joint properties, in the sense that a change of chaoticity of the neural system can induce a change in the stiffness of the springs in the joints. As a result the system is able to generate a large variety of patterns (stable, weakly stable, and unstable).

When the amplitude of the harmonics is too large, it negatively affects the groups formed in different regions of the body generating decoherence and destroying stable activity patterns. Note that the harmonic states are intrinsic to the coupling between neural, body, and environmental dynamics, and even if the spectral patterns seem complex, they should not be considered to be the outcome of yet another kind of neural noise.

We have previously suggested that behavioral changes are a complex function of the coupling between neural and body-environment dynamics. By using the Spectral Bifurcation Diagram we can now shed light on the

patterns of neural activity leading to such changes. For example for a level of chaoticity  $\alpha = 0.097477$  (that is, when the ring starts to roll) the power spectrum has a second harmonic which disappears in the interval  $[0.1, 0.13]$  (that is, when the system present difficulties to roll again) – see 1.4. The more harmonics there are, the more complex the behavior, despite preservation of coherence of behavior.

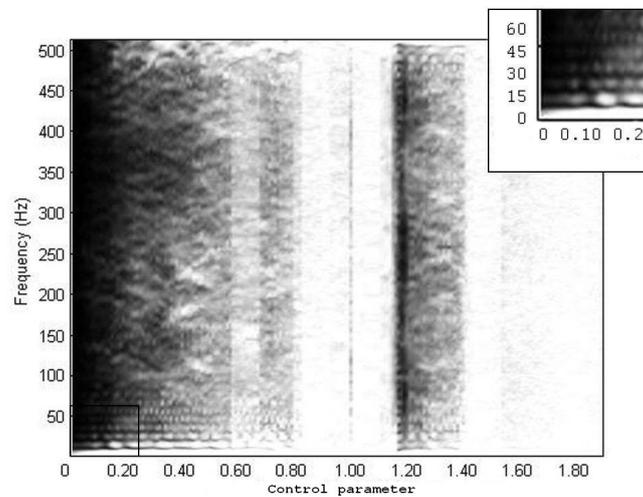


Fig. 1.4 Spectral Bifurcation Diagram. Inset shows spectral peaks for low values of neural chaoticity. The control parameter is  $\alpha$ .

### 1.5.2 Analysis 2: Neural coupling

We are interested in the spatio-temporal interaction patterns that emerge in the neural system. To get a better grasp on these patterns, we applied the Wavelet Transform to the activity of an arbitrarily selected chaotic unit for different values of the control parameter (see Figure 1.5). In the Wavelet space, the activities of the units disclose temporal correlations at different scales. The larger is the value of the control parameter, the higher is the complexity of the temporal patterns. Further analysis reveals a scale-free fractal-like structure in the neural activity and temporal coherence bridging multiple time scales. This result can be easily explained by considering the harmonics produced through feedback resonance. Long-range movements resulting from highly chaotic neural activity are composed of short-range movements triggered by lower values of chaoticity (Figure 1.5 c).

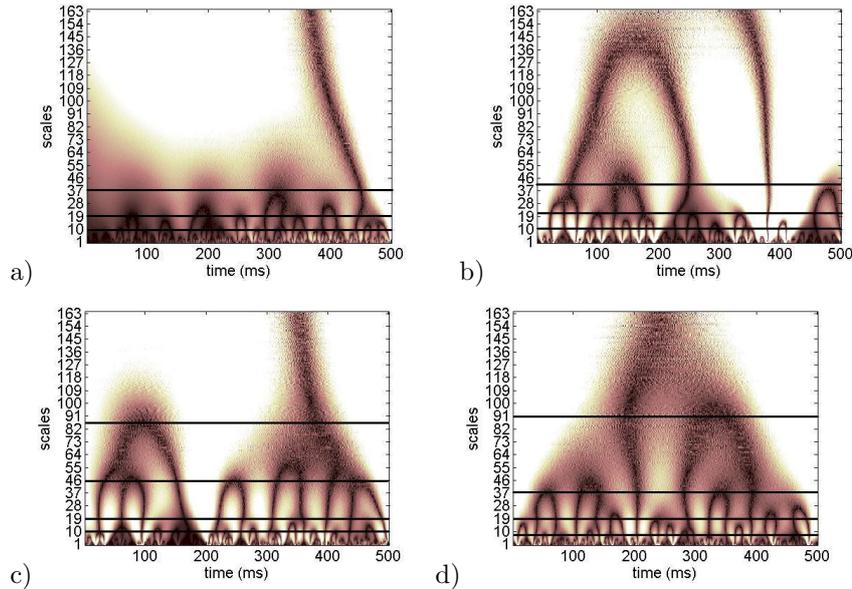


Fig. 1.5 Wavelet Transform. The spectro-temporal correlations between chaotic units are displayed. The control parameter  $\alpha$  varies linearly in the time interval 0-500 msec from (a) [0.0;0.5], (b) [0.5;1.0], (c) [1.0;1.5] and (d) [1.5;2.0].

One surprising effect is that the higher harmonics can dynamically alter the stiffness of the springs and thus their temporal responses. "Positive" resonance hardens the springs, and "negative" resonance softens them (see also [15]). We note also that the temporal scale of correlations seem to be quantified with respect to the control parameter (see horizontal lines in Figures 1.5 a-d). In other words, changes of "locomotion pattern" occur for specific values of neural chaoticity.

The Wavelet Bifurcation Diagrams of the neural activity for different values of the control parameter are shown in Figures 1.6 and 1.7. Through the mutual interactions between the neural system and the body, groups of neural units synchronize at multiple spatial scales (vertical axes of the figures). As in the case of the Wavelet Transform, the scale at which synchronization takes place and the type of emerging patterns depends on the amount of chaoticity in the neural system.

Interactions between neural system and body for low values of chaoticity form long-range correlations at a low spatial scale (rolling and balancing behavior). The "low-scale" spatio-temporal patterns correspond to disconnected short-range movements and the "high-scale" ones correspond to

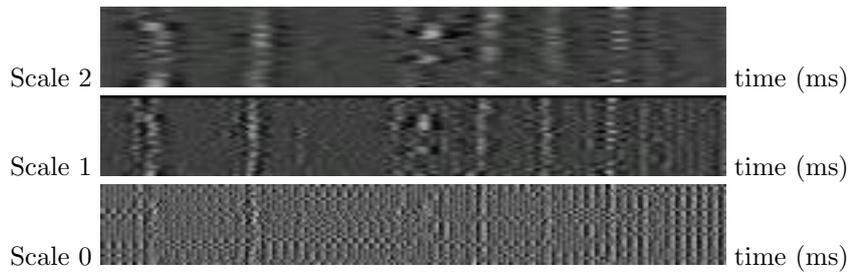


Fig. 1.6 Wavelet bifurcation diagram at different scales. In all three scale-plots, the horizontal axis denotes time (0-500 msec), and the vertical axis is the index of the chaotic unit.

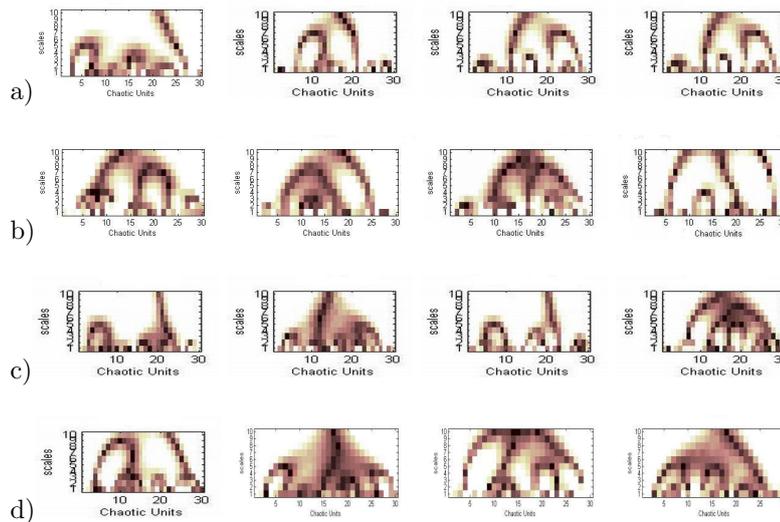


Fig. 1.7 Wavelet bifurcation diagram at different time instants. Snapshots of different neural configurations taken at different time instants for varying levels of chaoticity: a)  $\alpha = 0.5$ , b)  $\alpha = 1.0$ , c)  $\alpha = 1.5$ , d)  $\alpha = 1.9$ . The horizontal axis denotes the index of the chaotic unit, the vertical one is the scale (adimensional).

long-range movements. For higher chaoticity values, the same fractal-like spatio-temporal organizations are formed. Note that the control parameter is correlated to the complexity degree of the emergent behaviors. For low values of  $\alpha$ , stable activity groups of neurons are generated in the chaotic system. For higher values of  $\alpha$ , we can observe chaotic itinerancy in the system with an higher spatio-temporal complexity structure in the units. The groups are unstable and bifurcate to new transient configurations.

## 1.6 Discussion and Conclusion

In this paper, we introduced and discussed a novel framework for exploring action possibilities of complex mechanical systems. In particular, we studied (a) how chaotic neural activity can drive the exploration of movement patterns, and (b) how feedback resonance can be used to "tune into" particularly efficient movements. We also provided a set of tools to quantitatively measure the spatio-temporal organization of the neural system, and the stability of the emerging behavioral patterns.

We suggest that resonance plays a pivotal role for learning to control our bodies. Resonant states act as some kind of amplifier guiding the exploration and discovery of intrinsic modes of the body dynamics. One important side-result is the reduction of the number of degrees of freedom despite an increase in the overall complexity of the system. Another result is that resonance pushes the compliant actuators composing the body to dynamically alter their properties (e.g. stiffness) and to cooperate. In a sense, resonance also satisfies the principle of cheap design [9]. This principle states that when designing a system it is better to exploit physics and the dynamics of the system-environment interaction. Mapped onto our case study it means that resonance guarantees the emergence of energy-efficient movement patterns. As for learning or planning, this property can also be useful to understand when to increase or decrease the coupling between parts of the body, and to understand which parts have to be linked rather than testing all possible combinations. In addition, critical states (e.g. corresponding to unstable activity patterns or states where bifurcations occur) can also be identified and analyzed. We hypothesize that a mechanism of feedback resonance is responsible for combining unstable short-range patterns into stable long-range ones.

In future works we intend to implement our exploration model in a real robot situated in a dynamic environment. The robot will hopefully autonomously explore its body, and over time acquire a repertoire of complex, adaptive, and highly dynamic movements.

## 1.7 Acknowledgements

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