Synchronization: Adaptive Mechanism Linking Internal and External Dynamics

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Abstract

Synchronization is the dynamic adjustment of rhythms of oscillating systems. The question arises of whether the complex patterns emerging from the interaction of an embodied system's internal and external dynamics can be explained and quantified in terms of synchronization. Taking an information theoretical stance, we make the assumption that synchronization between coupled dynamical systems provides a powerful adaptive mechanism to receive, encode, and store external events into a system's internal dynamics. We illustrate our ideas with two simulation experiments: a system composed of two bidirectionally coupled chaotic map lattices, and an embodied system composed of a body and two coupled map lattices. We show how in both model systems a framework based on synchronization can smoothly integrate sensing and acting, and learn simple sensorimotor patterns without the need to postulate any *a priori* learning scheme. In the light of our results, we discuss the potential link between synchronization and the emergence and development of communication and imitation.

1. Introduction

Synchronization is the adjustment of two or more coupled systems to each other to give rise to some common dynamical behavior. Such behavior can result from either coupling the systems or by forcing them through a common external signal. Synchronization is such a pervasive phenomenon that it is studied in a wide range of research fields (Pikovsky et al., 2001, Strogatz, 2003). In brain science, appropriate synchronization is hypothesized to play an important role as a supporting mechanism for various modes of reciprocal (short and long-range) interaction between different brain regions, for perceptual and temporal binding, and as a necessary condition for the emergence of perceptual and conscious states (Roelfsema et al., 1997, Varela et al., 2001, Tsuda et al., 2004). In developmental psychology and psychophysics it is suggested that in infants the contingency (or, synchrony) between different sensory and motor channels may be a crucial aspect of the development of social cognition (Nadel et al., 2005, Prince and Hollich, 2005). Also, synchronization may be exploited to improve communication between humans and robots (Breazeal, 2002), or to enable a more efficient exploration of the sensorimotor space of embodied systems (Lungarella and Berthouze, 2002).

In this paper, we suggest to go one step further and use synchronization as a basic nonlinear mechanism to drive the learning of sensorimotor patterns and skills, and the development of communication and imitation. Synchronization may give a unitary, coherent, and hopefully formal view on how humans learn from and gradually mature with their own experiences. We start by giving some background on the notion of phase synchronization, a particular kind of synchronization which assesses the relation between the temporal structures of measured signals (typically, phases) regardless of their amplitudes. We then describe the methods used in this paper to detect synchronized states. By studying a system of coupled chaotic maps we show how while at synchronization such maps can exchange and store information, they are not able to do so when they are not synchronized. We proceed by studying an embodied system realized as a simulated humanoid robot, and show how externally imposed dynamics can affect its internal dynamics. Finally, we address the question of how to link synchronization and cognition, discuss the possibility to formalize semantic higher-level concepts relevant to psychology (e.g. memory, communication, and imitation) with the tools of physics, and point to some avenues worth exploring.

2. Phase Synchronization

In this section we first give an overview of synchronization and then introduce the methods used.

2.1 Background

Quite a few relevant applications of coupled nonlinear dynamical systems exists: excitable systems used for pattern detection (Baier and Muller, 2004), chaotic communication channels employed to transfer and encrypt information (Hayes et al., 1994), master-slave (driver-response) systems which can act as memories or "knowledge" predictors (Voss, 2000, Calvo et al., 2004). In all these applications, the coupling of two nonlinear systems (continuous or discrete) creates a communication channel along which information can flow (Hayes et al., 1994). It can be shown that the information flowing along the channel is proportional to the level of synchronization between the linked systems (Baptista and Kurths, 2005). For particular values of the coupling between the systems, the communication channel behaves poorly, that is, some parts of the information are transferred while other parts are lost or absorbed by the systems' subcomponents. What may be a drawback for communication can be of actual relevance from the point of view of embodied systems. As we hope to show in this paper, synchronization between coupled dynamical systems provides in fact a powerful mechanism to receive, encode, and embed external events into a system's internal dynamics. The balance between plasticity and stability of the system, and the ability to embed or to ignore external events is directly related to the amount of coupling between brain, body, and environment.

From a physical point of view, the coupling between two or more nonlinear (but active) systems (e.g. self-sustained oscillators) can be realized through modulation of the systems' phases and synchronization (Pecora and Carroll, 1990). Phase synchrony (PS) represents the counter-intuitive situation during which the phase of the coupled systems are locked, but their amplitudes are uncorrelated. In other words, the irregularities of the amplitudes can actually hide phase synchronization. While phase synchronized, the system's dynamics is integrated as well as differentiated. The "integrated" parts of the dynamics support the exchange of information by establishing communication channels between them, while the "differentiated" parts do not exchange any information. In this particular situation (denoted by $[\gamma_{PS-}, \gamma_{PS+}]$ in Fig. 1), both systems are flexibly coupled to each other and sustain a common rhythm.

Note that synchronization is not equivalent to resonance. Two coupled systems at resonance are not necessarily synchronized. Resonance is a response of a system that is non-active, i.e. demonstrates no oscillations without external forcing. Synchronization and resonance can be discerned by switching off or reducing the driving force and by observing if the rhythms disappear or not. In the case of resonance, some of the oscillations will decay after a transient. Thus, synchronization relies on the presence of weakly coupled active systems (e.g. self-sustained oscillators) which can function mostly (but not completely) independent from each other ($cf.$ Fig.1).

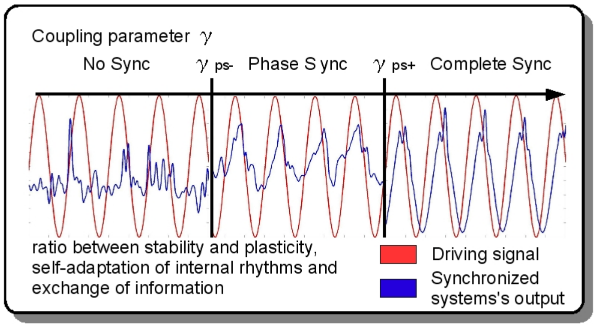

Figure 1: Phase synchronization and modulation.

2.2 Methods

Here we introduce the tools used to study synchronization, in particular, phase synchrony: a) phase synchronization index (Ψ); b) spectral bifurcation diagram (SBD); and c) wavelet bifurcation diagram (WBD).

a) Ψ: The method used to measure phase synchronization between two coupled systems is based on the concept of instantaneous phase (Rosenblum et al., 1996). The instantaneous phase can be calculated as $\phi = arctan\frac{x_H(t)}{x(t)}$ where $x_H(t)$ is the Hilbert transform of the narrow-band signal $x(t)$. The condition of 1:1 phase synchronization between two signals $x_1(t)$ and $x_2(t)$ is that the difference between their phases $\Delta\phi(t) = \phi_1(t) - \phi_2(t)$ stays bounded, whereas for unsynchronized states it corresponds to an unbounded growth of $\Delta\phi$(t).[1] In other words, two systems are synchronized if $|\Delta\phi(t)| < C$ where C is a constant. Given this definition of synchronization, we can define the phase synchronization index as:

$$\Psi = \sqrt{< cos\Delta\phi(t) >_t^2 + < sin\Delta\phi(t) >_t^2} \quad (1)$$

where $< . >_t$ denotes the temporal average. If the signals $x_1(t)$ and $x_2(t)$ are phase synchronized $\Psi = 1$ (which indicates a constant phase difference). For a uniformly distributed phase difference (that is, no synchronization) $\Psi = 0$.

b) SBD: In order to investigate qualitative changes of dynamics in high-dimensional systems, we use the spectral bifurcation diagram (Orrel and Smith, 2003). Essentially, SBD displays the power density spectrum of multiple

[1] A more general definition includes rational relationship between the phases: $|n\phi_1 - m\phi_2|$ where n and m are arbitrary integers.

system variables as a function of a system control parameter (e.g. force, temperature, coupling strength) (the power spectra of the individual variables are superposed). This method allows identification of resonant states characterized by sharp frequency components, chaotic states having rather broad power spectra, as well as bifurcations, that is, qualitative changes in the system's behavior (phase transitions from one state attractor to another).

c) WBD: Because we are interested in understanding the spatial correlations between coupled dynamical systems (e.g. neural units), as well as correlations at different spatio-temporal scales, we also used the wavelet bifurcation diagram (Pitti et al., 2006). This measure based on the wavelet transform spans frequency, time, and the index of the time series analyzed, and allows visualization of spatio-temporal patterns, bifurcations and chaotic itinerancy at different scales for a high dimensional system. It can be conceptualized as a multi-resolution tool for the analysis of multi-variate time series.

3. Experimental Studies

To illustrate our ideas we investigate two model systems: a system composed of two bidirectionally coupled chaotic map lattices (CML), and an embodied system consisting of a body which is bidirectionally coupled to two CMLs. A CML is a dynamical system with discrete time, discrete space, but a continuous state. It consists of L coupled dynamical units which typically exhibit nonlinear dynamics. In this paper for the dynamical units we choose the logistic map:

$$f(z) = 1 - \alpha z^2. \tag{2}$$

The behavior (chaoticity) of the chaotic map $f(z)$ is controlled by the parameter $\alpha \in [0, 2]$ (identical for all units). The amplitude of the map varies between $[-1, 1]$. All units are connected to their two nearest neighbors and to an external input (e.g. sensor feedback) whose amplitude is normalized between $[-1, 1]$ by a coupling $\gamma \in [0, 1]$ (external coupling).

3.1 Coupled chaotic maps

As a first example, we study phase synchronization in two bidirectionally coupled CMLs $x_n(i)$ and $y_n(i)$ consisting of $L = 32$ chaotic units each. The coupled system's behavior is expressed as:

$$\begin{aligned} x_n(i+1) &= (1-\gamma)\, f(x_n(i)) + \tfrac{\gamma}{2}\,(x_n^{int}(i) + x_n^{ext}(i)) \\ y_n(i+1) &= (1-\gamma)\, f(y_n(i)) + \tfrac{\gamma}{2}\,(y_n^{int}(i) + y_n^{ext}(i)) \end{aligned}$$

with

$$\begin{aligned} x_n^{int}(i) &= (x_{n-1}(i) + x_{n+1}(i))/2 \\ y_n^{int}(i) &= (y_{n-1}(i) + y_{n+1}(i))/2 \\ x_n^{ext}(i) &= y_n(i) \\ y_n^{ext}(i) &= x_n(i) \end{aligned} \tag{3}$$

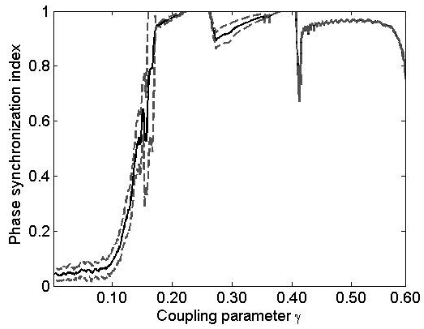

Figure 2: Phase synchronization index CML 1 - CML 2. Black continuous line: average of Ψ over 32 neural units; gray dashed line: standard deviation.

where n is a discrete time step and $i \in [1, 32]$ is the index of the i^{th} chaotic unit; x^{int}, y^{int}, and x^{ext}, y^{ext} are the internal and external couplings respectively.

Depending on the coupling parameter γ, the two coupled CMLs may or may not produce sustained oscillations. In this experiment we vary the coupling parameter γ between $[0, 0.6]$ and observe the coupled system's behavior after perturbing the first CML with an arbitrary signal $s_n(i) \in [-1, 1]$ (the second CML $y_n(i)$ is not externally perturbed). Accordingly, the equation governing the dynamics of the first CML is modified as follows:

$$x_n^{ext}(i) = (y_n(i) + s_n(i))/2. \tag{4}$$

The experiments are conducted with $N = 100000$ samples. By analyzing the outcome of the experiments in the spectral domain and in the phase space, we can better understand the structure of the interactions and their time evolution. The phase synchronization index Ψ between the two coupled CMLs is plotted in Fig. 2. The black continuous line denotes the average of Ψ calculated over all 32 units, the gray dashed lines indicate the standard deviation. At phase synchronization (PS) the standard deviation is rather large indicating flexibility of the system to assume different states. The evolution over time of the probability distribution of a representative neural unit for different values of γ is displayed in Fig. 3. As evident from both figures, for this particular system, PS occurs in two intervals for γ in $[0.15, 0.18]$ and $[0.27, 0.37]$ characterized by a high level of synchronization and a high variability. In the subsequent analysis, we focus our study on the interval $[0.15, 0.18]$.

Below a certain value of the external coupling constant $\gamma < \gamma_{PS^-} = 0.15$, $\Psi \approx 0.5$, and we do not observe any synchronization between the two CMLs (Fig. 2). Both systems are uncoupled, and their dynamics is unconstrained and independent from each other (Fig. 3 a). The information fed to the system through the external perturbation s_n is absorbed by the first CML's dynamics and there is no exchange of information between the two maps.

At PS, for $\gamma = \gamma_{PS^-}$ the two systems begin to interact. Both systems are weakly coupled and synchronized and $\Psi \approx 0.5$. The two systems form a weak communication channel and their phases assume values so that information can transit from one system to the other. External information can now flow through the first CML and, at least partially, influence the state of the second CML (see Figs. 3 b and c). Because the two systems reciprocally affect each other, some information can be stored in the coupled system's dynamics.

For $\gamma \in [\gamma_{PS^-}, \gamma_{PS^+}]$, the amount of information exchanged is proportional to the coupling γ. As indicated by the probability distributions, the difference between parts of the dynamics of the two systems increases (differentiation) while at the same time the external information tends to intergrate other parts (integration). At PS the systems are able to trade-off plasticity and stability, that is, they are able to react adaptively to external perturbations while maintaining their own intrinsic dynamics.

For $\gamma > \gamma_{PS^+} = 0.23$, the dynamics of both systems are fully coupled and they behave as one unique system. The two systems are completely synchronized and information is transmitted with a higher accuracy and with less loss through the established channel and less flexibility (Fig. 3 e).

These results suggest that phase synchronized systems are characterized by stability and plasticity. They also explain how information is exchanged and stored independently of the internal structure of the coupled systems and of the nature of the information.

3.2 Embodied system

Our second experimental system is a simulated humanoid robot realized with the Novodex graphics engine of Aegia ("http://www.aegia.com"). We use four of its mechanical degrees of freedom (two elbows and two knees) all of which are connected to two coupled CMLs using Eq. 3 (see Fig. 4). Both CMLs consist of $L = 32$ coupled logistic maps ($\alpha = 2.0$). The first lattice receives inputs from the angular sensors (chaotic units $i = 5$, 13, 21 and 29) forming the system "body-CML 1". The units with the same index of the second lattice (with outputs normalized between [-1,1]) are connected to the actuators, forming the system "CML 2-body". To extract the phase embedded in the chaotic signal the output is filtered with a mexican-hat wavelet expressed as: $F_i^{body}(n) = mexh_a(y_i(n))$, where $mexh_a(x) = \frac{2}{\sqrt{3a}}\pi^{-1/4}(1-x^2)e^{\frac{-x^2}{2}}$, with a being the scale of the wavelet ($a = 64$ for our application). Our model system consists of two active oscillating systems ("body-CML 1" and "CML 2-body") and hence satisfies the minimal requirement for displaying phase synchronized states. The sampling time is $2.4\,ms$.

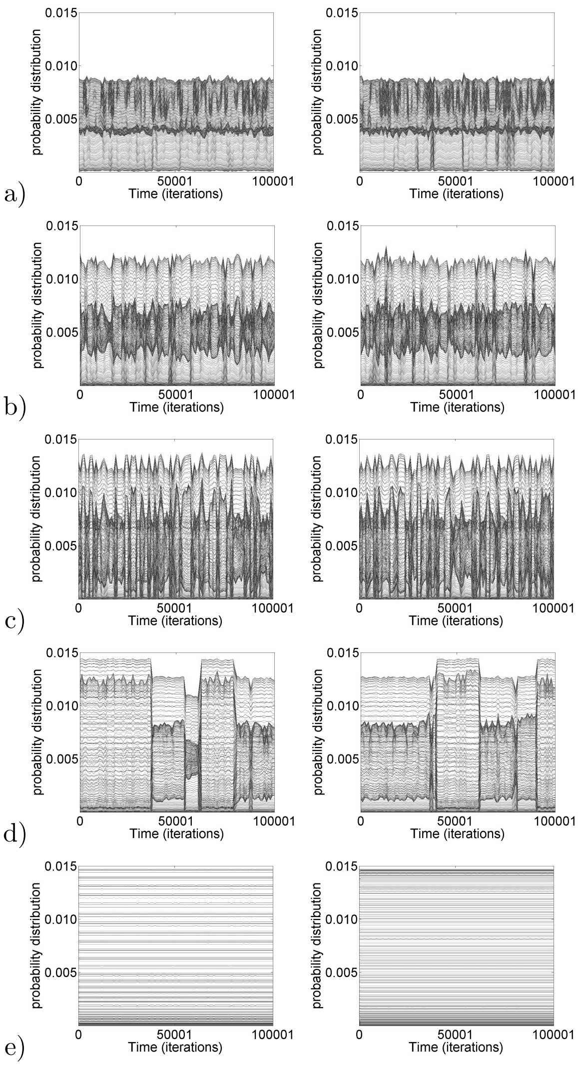

Figure 3: Probability distribution as a function of time for representative neural unit. Left column: CML 1; right column: CML 2. Increasing the coupling γ increases the influence of external input on systems' internal dynamics. The coupling constants are: $\gamma = 0.10$ (a); $\gamma = \gamma_{PS^-} = 0.15$ (b); $\gamma = 0.16$ (c); $\gamma = \gamma_{PS^+} = 0.18$ (d); $\gamma = 0.23$ (e).

3.2.1 Varying the coupling parameter

As in the previous section, we analyze the synchronization level between the two CMLs by varying the coupling parameter γ. We plot the phase synchronization index and the corresponding spectral bifurcation diagram of the CMLs (Fig. 5) for $\gamma \in [0, 0.60]$. For this model system, phase synchronization occurs for several intervals γ – specifically $[0.15, 0.18]$, $[0.23, 0.27]$ and $[0.28, 0.41]$. In the subsequent analysis, we focus on the interval $[0.15, 0.18]$.

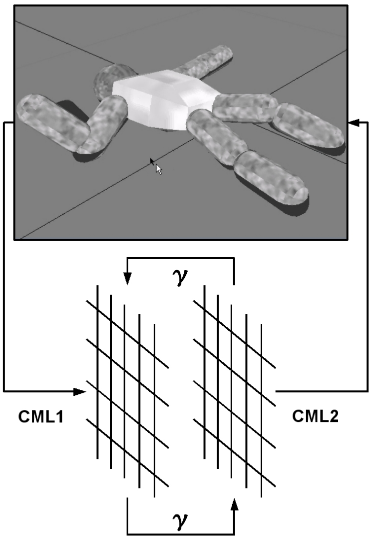

Figure 4: Outline of our embodied model system. "Body-CML 1" and "CML 2-body" form two self-sustained oscillators.

For $\gamma < \gamma_{PS^-} = 0.15$, the chaotic units composing the CMLs are characterized by broadband spectral distribution with no well-pronounced peaks. We observe rapid movements of arms and legs with small amplitudes. When perturbed (e.g. through touch), the body's limbs oppose only weak resistance; the external dynamics has no influence on the internal dynamics. There is some interaction but definitely no synchronization between the body and the neural dynamics, and each system acts independently.

For $\gamma \in [0.15, 0.18]$, PS occurs between the body and the neural system. The spectral dynamics is now narrow-band and characterized by a few significant peaks (i.e. a dimension reduction of the global system dynamics occurs). Both dynamics are now correlated and mutually integrated through the phase (condition of phase-locking). In analogy with the information theoretic framework discussed previously, a communication channel between body and neural system is established through which information can be exchanged. In this situation, touching or manipulating the limbs externally affects the internal dynamics of the system. After releasing the force applied to the limbs, the system is for some time able to reproduce the previously imposed movement (see also the following subsection). Because the two CMLs are synchronized, it is possible to link internal and external dynamics, and embed externally imposed sensorimotor patterns through phase modulation.

Beyond phase synchronization ($0.23 > \gamma > 0.18 = \gamma_{PS^+}$), the level of synchronization between the two CMLs increases and their dynamics becomes more uniform. Information is exchanged between the two systems but is not embedded.

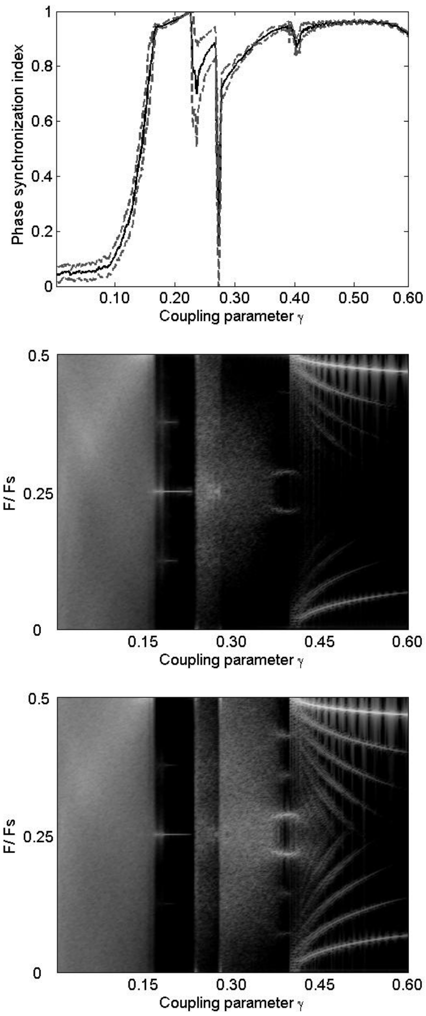

Figure 5: Embodied setup – variable γ. Top: Phase synchronization index vs. coupling γ for CML 1 - CML 2. Center: Spectral bifurcation diagram for CML 1. Bottom: Spectral bifurcation diagram for CML 2.

3.2.2 At phase synchronization

In this section, we analyze the behavior of the embodied system for coupling parameter $\gamma = 0.15$, that is, body, CML 1 and CML 2 are weakly coupled and phase synchronized.

For $\gamma = 0.15$ the system is active, sensitive to external oscillations, and able to embed and sustain them in its internal dynamics. By forcing the body's limbs to follow particular movements, we can influence the systems's internal dynamics and store externally applied movements. As a result, the system can partially learn and reproduce the imposed movements. If the movement is blocked (e.g. by an obstacle or an external force), then the behavior flexibly adapts to the new boundary condition. To analyze the internal organization of the systems, we plot the phase synchronization index Ψ for the system composed of CML 1 - CML 2 (Fig. 6 top). The activity of the joint angle sensor in the elbow is displayed in Fig. 6 center (the dashed line is the imposed pattern, the continuous line is the pattern at the output of the system). To show that patterns are actually stored and reproduced, we also plot the output for the case $\gamma = 0.40$ (strong synchronization; Fig. 6 bottom).

The large variance of Ψ around 0.5 suggests readiness of the system to react to different sensorimotor patterns – it allows exchanges of information and the mutual adaptation of rhythms of CML 1 and CML 2. The oscillatory fluctuations of Ψ point to periods of synchronization (high values), synchronization breaks (low values), and resynchronization. To better understand the underlying dynamics, we compute the wavelet bifurcation diagram for the two CMLs (Fig. 7). Low scales (high frequencies) emphasize the local variabilities while high scales (low frequencies) express the global structure of the system. We can also observe chaotic itinerancy in the two CMLs for time scales higher than 3 (Fig. 7 c). For these time scales the dynamics of the clusters are similar, that is, synchronization occurs but is "itinerant" and characterized by instabilities which produce a transitory behavior between attractor states, and the appearance and disappearance of phase-related synchronizations. The near-zero Lyapunov exponents of the time series indicate that the system is effectively in a state of chaotic itinerancy and phase synchronization (Fig. 8) (Tsuda et al., 2004).

The synchronization break ($t = 6500\,ms$; Fig. 7) is explained as follows: the externally imposed movement is "forced" upon the internal systems and the previous patterns are (at least partially) destroyed. When resynchronization occurs, patches of synchronous but decaying neural activity can be observed.

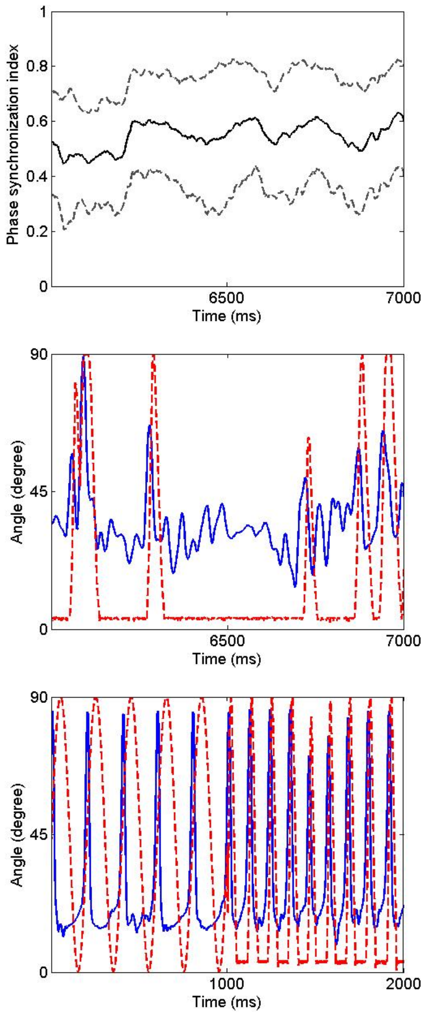

Figure 6: Embodied setup – fixed γ. Top: Phase synchronization index vs. time; coupling between CML 1 and CML 2 fixed to $\gamma = 0.15$. Center: Input-output joint angle activity (elbow) at phase synchronization ($\gamma = 0.15$). Bottom: Input-output angle activity (elbow) for highly synchronized systems ($\gamma = 0.40$).

4. Discussion

Synchronization is the adaptation of rhythms of coupled oscillators. Using a disembodied and an embodied setup we have shown how at synchronization individual (internal) and environmental (external) dynamics can be coupled more efficiently, e.g. externally imposed sensorimotor patterns are transferred and stored more easily internally. This result points to an important role of phase synchronization for selective attentional processes: selection of sensorimotor patterns is achieved dynamically through modulation of phases between the coupled systems. Moreover, depending on the system's current dynamics and on the amount of coupling between internal and external dynamics, filtered external signals can persist for more or less time (patches of synchronous, persistent, but decaying neural activity are shown in Fig. 7) and the neural system can function as either a short-term or a long-term memory. Such persistence is related to the idea of perceptual continuity (Hock et al., 2003), that is, of coherent and stable perception despite disruption by environmental perturbations or noise. The interaction between internal and external dynamics is reciprocal. If the brain and body-environment are adequately coupled, stable behavioral patterns such as walking or running could emerge despite perturbations or noisy signals (because (PS) occurs regardless of the amplitude of the signal), and yield complex emergent dynamics (because (PS) is a general mechanism independent of the movement patterns's complexity).

Phase synchronization between the neural units and phase transitions are also exhibited by several models of memory dynamics (Tsuda et al., 2004, Raffone and van Leeuwen, 2003). Adaptive Resonance Theory is also related to the notion of synchro-

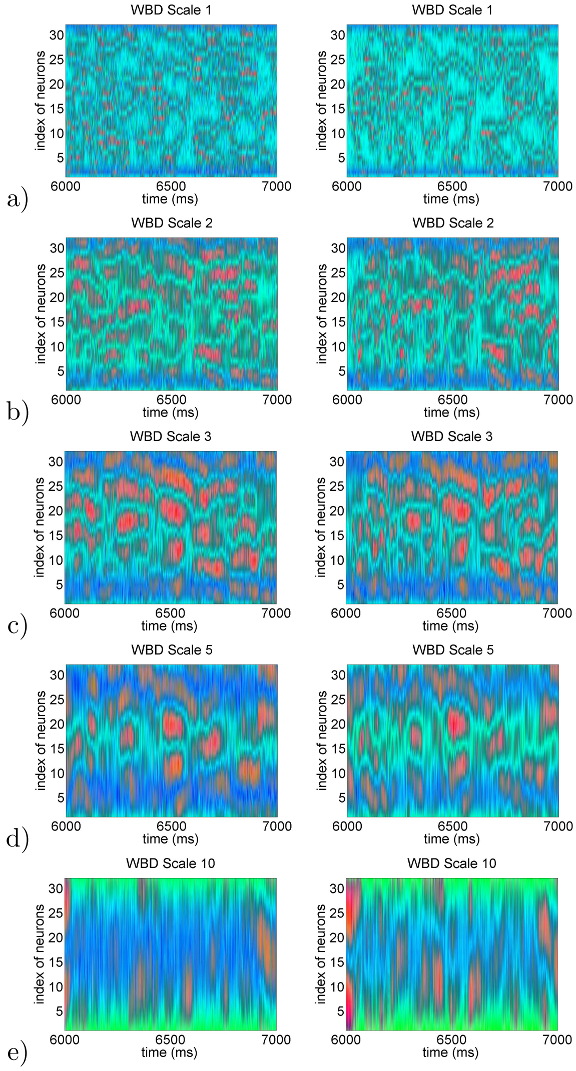

Figure 7: Wavelet bifurcation diagrams. Left column: CML 1; right column: CML 2. The scales are s=1 (a), s=2 (b), s=3 (c), s=5 (d), s=10 (e).

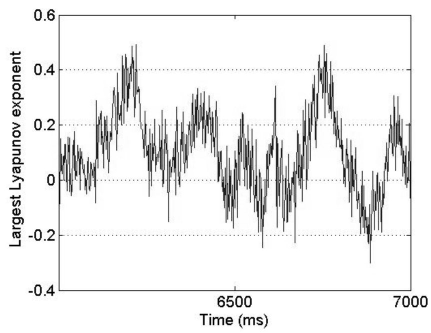

Figure 8: Fluctuations of the largest Lyapunov exponent.

nization (Grossberg, 2005). Specifically, at phase synchronization there is a trade-off between stability and plasticity, that is, the neural system learns quickly novel patterns (plasticity) without forgetting catastrophically past knowledge (stability or persistence). Grossberg further suggests that all conscious states in the brain are resonant states, and that these resonant states trigger learning of sensory and cognitive representations. It might be worthwhile to generalize these results in an embodied viewpoint and justify them from a physical and information theoretical point of view (as attempted in this paper).

As synchronization is the physically-based adaptation of systems rhythms, it can also be conceptualized as an adaptive learning mechanism which neither makes any *a priori* assumptions nor uses any specific learning scheme. We hypothesize that synchronization may actually allow a reinterpretation of Hebb's law: "Neurons that fire together, wire together" may be re-formulated as "neurons that synchronize, wire together". We suggest that two synchronized nonlinear oscillators, even if only weakly coupled (that is, potentially physically disconnected), can actually "wire" together, i.e. exchange information.

In the context of social interaction, the notion of synchronization (or synchrony, resonance, social entrainment or regulation) has been already introduced as a concept (e.g. (Breazeal, 2002, Nadel et al., 2005)). We hypothesize that phase synchronization may also shed some light on the mechanisms underlying social interaction, and may help formalize and explain high-level semantic concepts such as self-agency and resonance, coupling and autonomy (e.g. language, communication or their dysfunctionment like autism).

It is also interesting to consider coupled systems in the context of cause-effect relationships. Since the first observation of anticipation of synchronization in a physical system (Voss, 2000), it has been shown that at PS, one of the two coupled systems can anticipate the state of the other (Calvo et al., 2004). As the systems act together, it follows that the effect may actually precede its cause. From this result, we can imply that for a particular system it might be necessary to revise the notion of unidirectional relationship between causality and time (Prigogine, 1980). It also follows that synchronization can stand for the design of a physically-based and dynamical predictive model and for a memory model.

Before concluding, we would like to point out that phase as used here is a dubious concept for discrete time systems and can in general only be introduced for continuous dynamical systems (Rosenblum et al., 2001). The reason is that phase can be only defined for systems having zero Lyapunov exponents. As evident from Fig. 8, the results obtained with coupled logistic maps exhibited intermittent near-zero Lyapunov exponents which are characteristic of chaotic itinerancy but which weakly violate the strict definition of phase given in (Rosenblum et al., 2001). Recent work on phase synchronization, however, seems to confirm our observations, and that is, phase can be also defined for discrete systems (at least in particular instances) (Koronovski et al., 2005). Our results seem to indicate that the framework described in this paper can be actually used for discrete time systems.

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